

SAMPLE QUESTION PAPER - 1

Subject – Mathematics (041)

Class XII

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

1. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is [1]
(A) 2 (B) 1
(C) -1 (D) 0
2. The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is [1]
(A) 10 (B) -7
(C) 7 (D) 8
3. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to: [1]
(A) -8 only (B) 64
(C) 8 only (D) 8 or -8
4. If $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$, then A^2 is: [1]
(A) 27A (B) 3A
(C) 2A (D) I
5. The Cartesian equations of a line are $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$. Its vector equation is [1]
(A) $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ (B) $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + \lambda(\hat{i} - 2\hat{j} + 5\hat{k})$
(C) $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 4\hat{k})$ (D) $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - 4\hat{k})$
6. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is: [1]
(A) e^x (B) $\frac{x}{e^x}$
(C) $\frac{e^x}{x}$ (D) xe^x

7. A Linear Programming Problem is as follows: [1]
 Maximize / Minimize objective function $Z = 2x - y + 5$
 Subject to the constraints
 $3x + 4y \leq 60$
 $x + 3y \leq 30$
 $x \geq 0, y \geq 0$
 In the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0, 0), then which of the following is true?
 (A) Minimum value of Z is -5 (B) At two corner points, value of Z are equal
 (C) Maximum value of Z is 40 (D) Difference of maximum and minimum values of Z is 35
8. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to [1]
 (A) -1 (B) $\frac{1}{\sqrt{2}}$
 (C) $\sqrt{2}$ (D) 1
9. $\int \frac{1+\tan x}{1-\tan x} dx$ is equal to: [1]
 (A) $\sec^2\left(\frac{\pi}{4}-x\right)+C$ (B) $\sec^2\left(\frac{\pi}{4}+x\right)+C$
 (C) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+C$ (D) $\log\left|\sec\left(\frac{\pi}{4}+x\right)\right|+C$
10. Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2023)^x$ is equal to: [1]
 (A) 2023 (B) $(2023)^2$
 (C) $\frac{1}{2023}$ (D) 1
11. The corner points of the feasible region determined by the following system of linear inequalities:
 $2x + y \leq 10, x + 3y \leq 15, x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q \geq 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is [1]
 (A) $p = 3q$ (B) $q = 3p$
 (C) $p = q$ (D) $p = 2q$
12. A unit vector $\hat{a}$ makes equal but acute angle on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is [1]
 (A) $\frac{3}{5\sqrt{3}}$ (B) $\frac{11}{15}$
 (C) $\frac{4}{5}$ (D) $\frac{11}{5\sqrt{3}}$
13. Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If $AX = B$, then X is equal to [1]
 (A) $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ (C) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

14. If for two events A and B, $P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to [1]

- (A) $\frac{1}{2}$ (B) $\frac{2}{5}$
(C) $\frac{2}{3}$ (D) $\frac{3}{5}$

15. Find the particular solution for $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$ when $x = 1$ [1]

- (A) $y = \frac{2x}{1 - \log|x|}$ ($x \neq 0, x \neq e$) (B) $y = \frac{3x}{1 - \log|x|}$ ($x \neq 0, x \neq e$)
(C) $y = \frac{2x}{1 - \log|x|}$ ($x \neq 0, x \neq e$) (D) $y = \frac{5x}{1 - \log|x|}$ ($x \neq 0, x \neq e$)

16. If $\vec{a}, \vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is: [1]

- (A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$
(C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{6}$

17. If $y = \left(1 + \frac{1}{x}\right)^x$, then $\frac{dy}{dx} =$ [1]

- (A) $\left(1 + \frac{1}{x}\right)^x \log\left(1 + \frac{1}{x}\right)$ (B) $\left(x + \frac{1}{x}\right)^x \left\{\log\left(1 + \frac{1}{x}\right) + \frac{1}{x+1}\right\}$
(C) $\left(1 + \frac{1}{x}\right)^x \left\{\log\left(1 + \frac{1}{x}\right) - \frac{1}{x+1}\right\}$ (D) $\left(x + \frac{1}{x}\right)^x \left\{\log(x+1) - \frac{x}{x+1}\right\}$

18. If the line $\frac{x-2}{2k} = \frac{y-3}{3} = \frac{z+2}{-1}$ and $\frac{x-2}{8} = \frac{y-3}{6} = \frac{z+2}{-2}$ are parallel, value of k is [1]

- (A) -2 (B) 2
(C) $\frac{1}{2}$ (D) 4

19. **Assertion (A)** : If $3 \leq x \leq 10$ and $5 \leq y \leq 15$, then minimum value of $\left(\frac{x}{y}\right)$ is 2. [1]

Reason (R) : If $3 \leq x \leq 10$ and $5 \leq y \leq 15$, then minimum value of $\left(\frac{x}{y}\right)$ is $\frac{1}{5}$.

- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

20. If R is the relation in the set $A = \{1, 2, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, [1]

Assertion (A) : R is an equivalence relation.

Reason (R) : All elements of $\{1, 3, 5\}$ are related to all elements of $\{2, 4\}$.

- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.

- (C) A is true but R is false.
(D) AA is false but R is true.

SECTION B

21. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. [2]

OR

Write the interval for the principal value of function and draw its graph: $\sin^{-1}x$

22. The volume of a cube is increasing at the rate of $7 \text{ cm}^3/\text{sec}$. How fast is its surface area increasing at the instant when the length of an edge of the cube is 12 cm? [2]
23. Find the maximum and minimum values of $2x^3 - 24x + 107$ on the interval $[-3, 3]$. [2]

OR

Show that $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is an increasing function on $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$.

24. Evaluate : $\int \frac{(3x+5)}{(x^3-x^2-x+1)} dx$ [2]
25. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3, 0) \cup (0, 3)$. [2]

SECTION C

26. Evaluate : $\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$ [3]
27. Bag I contains 3 white and 4 black balls, while Bag II contains 5 white and 3 black balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be white. Find the probability that the transferred ball is also white. [3]
28. Evaluate : $\int_0^{1/\sqrt{2}} \frac{\sin^{-1}x}{(1-x^2)^{3/2}} dx$ [3]

OR

Find $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$.

29. Find particular solution of differential equation $(1+y^2) + \left(x - e^{\tan^{-1}y}\right) \frac{dy}{dx} = 0$, given that $y = 0$ when $x = 1$. [3]

OR

Solve : $\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$

30. Show that the solution set of the linear constraints is empty : $x - 2y \geq 0$, $2x - y \leq -2$, $x \geq 0$ and $y \geq 0$ [3]

OR

Solve the following LPP by graphical method:

Minimize $Z = 20x + 10y$

Subject to

$$x + 2y \leq 40$$

$$3x + y \geq 30$$

$$4x + 3y \geq 60$$

$$\text{and } x, y \geq 0$$

31. If $x = a \sin 2t (1 + \cos 2t)$ and $y = b \cos 2t (1 - \cos 2t)$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. [3]

SECTION D

32. Using integration, find the area of the region in the first quadrant enclosed by the Y-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. [5]
33. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Write all the equivalence classes of R . [5]

OR

Show that the function $f : \mathbb{R} \rightarrow \{x \in \mathbb{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$ is one-one and onto function.

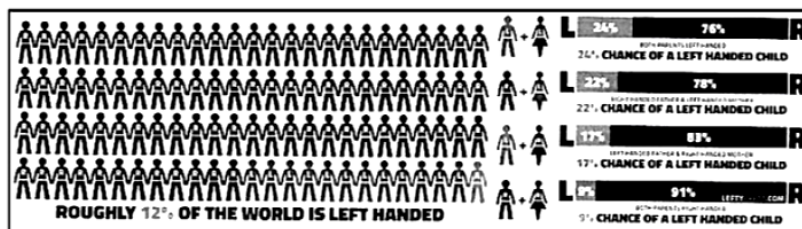
34. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs2200.
School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs. 1200, using matrices, find the award money for each value. [5]

35. Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z-3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Find the image of the point $(0, 2, 3)$ in the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ [5]

SECTION E

36. Recent studies suggest that roughly 12% of the world population is left handed. [4]



Depending upon the parents, the chances of having a left handed child are as follows:

- When both father and mother are left handed: Chances of left handed child is 24%.
- When father is right handed and mother is left handed: Chances of left handed child is 22%.
- When father is left handed and mother is right handed: Chances of left handed child is 17%.
- When both father and mother are right handed: Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

i. Find $P\left(\frac{L}{C}\right)$ (1)

ii. Find $P\left(\frac{L}{A}\right)$ (1)

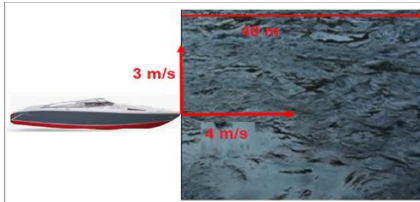
iii. Find $P\left(\frac{A}{L}\right)$ (2)

OR

Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

(2)

37. Read the following text carefully and answer the questions that follow: [4]
Once Ramesh was going to his native place at a village near Agra. From Delhi and Agra he went by flight, In the way, there was a river. Ramesh reached the river by taxi. Then Ramesh used a boat for crossing the river. The boat heads directly across the river 40 m wide at 4 m/s. The current was flowing downstream at 3 m/s.



- What is the resultant velocity of the boat? (1)
- How much time does it take the boat to cross the river? (1)
- How far downstream is the boat when it reaches the other side? (2)

OR

If speeds of boat and current were 1.5 m/s and 2.0 m/s then what will be resultant velocity? (2)

38. Read the following text carefully and answer the questions that follow: [4]
Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.



- Find the volume of the open box formed by folding up the cutting each corner with x cm. (1)
- Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? (1)
- Verify that volume of the box is maximum at $x = 3$ cm by second derivative test? (2)

OR

Find the maximum volume of the box.

**** End ****

For Solution
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the code

